Implicit Priorities of Incoming College Freshman

by

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1 Abstract

We apply the Analytic Hierarchy Process, an analytic decision-making tool, in a realistic simulation for finding the ranking of priorities. In this paper, we are specifically concerned with the implicit priorities of incoming college freshman. By demonstrating the Analytic Hierarchy Process in specific examples and in the simulation, we were able to show that the Analytic Hierarchy Process is an effective and feasible tool to use in further research on the implicit priorities of incoming college freshman.

2 Introduction

A common trait that incoming college freshman tend to possess is having difficulty in understanding what exactly their priorities are. Although college freshman are often given the advice to manage their time wisely, time management is an art that must be perfected over time and comes with maturity. When initiating a college career, some of the most important and most difficult decisions include prioritizing between having a social life, maintaining a high academic standing, and growing intellectually. It may prove to be very challenging for an incoming student to decide if gaining intellectual ability is more important than making new friends and enjoying life during their college career, and if so, by how much. It is especially more difficult as you add in other priorities that may come in to play such as gaining significant experience to train for a future career, understanding exact career goals, and developing important skills such as writing, public speaking, and data manipulation. Trying to compare every possible priority all at once may prove to be a very confusing, biased, and uncertain method of coming to a final ranking of priorities. However, when applying a more analytical approach, such as the Analytic Hierarchy Process, an optimal priority ranking may be made with a much higher level of confidence and with a significantly less amount of confusion.

Introduced in the mid 1970's by Thomas L. Saaty and later revised in 1983 by Belton and Gear, the Analytic Hierarchy Process quickly became one of the most efficient and popular decision making tools of its kind [2]. The Analytic Hierarchy Process allows an individual to compare two priorities at a time to minimize the bias in the individual's decisions [5]. As one can imagine, in most cases it is significantly easier to compare only two priorities at once rather than comparing one priority to several other priorities at once. When implementing the Analytic Hierarchy Process, one will rate the level of importance of each priority, two at a time, and will place those ratings into what is known as a pairwise-comparison matrix. For example, an individual is planning on purchasing a vehicle and has the following three options:

- SUV
- Sports Car
- Minivan

The only concerns that individual has in determining which vehicle to purchase are the following:

- Gas Mileage
- Style
- Convenience

Let's assume that that individual believes that gas mileage is 3 times as important as style, convenience is 2 times as important as gas mileage, and style is 6 times as important as convenience. Then the pairwise-comparison matrix will appear as the following:

$$A = \begin{pmatrix} 1 & 3 & 1/2 \\ 1/3 & 1 & 6 \\ 2 & 1/6 & 1 \end{pmatrix}$$

Note that the first, second, and third rows correspond to gas mileage, style, and convenience, respectively; and the first, second, and third columns also correspond to gas mileage, style, and convenience, respectively. Since the individual believes that gas mileage is 3 times as important as style, there will be a 3 at $a_{1,2}$. To determine the relative importance of style compared to gas mileage, we simply invert the importand of gas mileage compared to style. Therefore $a_{2,1} = 1/3$.

The next step of the Analytic Hierarchy Process would be to find the eigenvector that corresponds with the maximum lambda value that will actually give the rankings of the priorities. It may be important to note the following theorem: *For a given positive matrix, A, the only positive vector $\vec{x}$ and only positive constant c that satisfy $A\vec{x} = c\vec{x}$, is a vector $\vec{x}$ that is a positive multiple of the principle eigenvector of A, and the only such c is the principle eigenvalue of A.* This result, known as the Perron-Frobenius Theorem, appears with proof in Gantmacher's *Applications of the Theory of Matrices*[7]. Consider the following matrix equation:

$$\begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ 1/a_{12} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/a_{1n} & 1/a_{2n} \cdots & 1 & \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \lambda_{max} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

The positive eigenvector gives us the final ranking of each element in the pairwise comparison matrix because it is essentially the underlying scale for each priority [6]. Another thing that should be mentioned about the Analytic Hierarchy Process is that if the pairwise comparison matrix is completely consistent $(\forall(i,j,k) : a_{ij}a_{jk} = a_{ik})$ if and only if the characteristic polynomial is $\lambda^n - n\lambda^{n-1}$. In layman's terms, if a pairwise-comparison matrix is completely consistent, implementing the analytic hierarchy process will not be beneficial because one could simply rank their priorities directly. As this essentially never happens in practice, there is an extensive literature for improving priority matrix consistency; see, e.g., Alonso and Lamata, "Consistency in the Analytic Hierarchy Process: A New Approach" [8].

For the example used above, the eigenvector is:

$$\begin{bmatrix} 0.6228 \\ 0.6854 \\ 0.3772 \end{bmatrix}$$

Recall that the first, second, and third rows correspond to gas mileage, style, and convenience, respectively. Since the second row has the highest value in the eigenvector at 0.6854, style will be ranked as the top priority. Gas mileage will be ranked second and convenience third. To continue the Analytic Hierarchy Process, the values in the eigenvector will serve as weights for each priority. The individual could then do the same process by comparing each vehicle in terms of each priority, and then finding the total weighted average of each vehicle's eigenvector value. This will give the final ranking of decisions [1].

Due to the simplicity and overall effectiveness of the Analytic Hierarchy Process, the decision making technique has found its way into making various types of decisions in corporations, government, and fields of engineering [4]. Whether making a rather straightforward decision, like deciding on a car to purchase, or making a much more complicated decision, the Analytic Hierarchy Process has proven its success time and time again in reducing opinion bias, simplifying the decision-making process, and optimizing decisions.

3 Methods

3.1 Initial Project

The initial goal of the project was to determine the implicit priorities of incoming college freshman. By creating a survey in Amazon's Mechanical Turk, we would be able to anonymously collect information directly from incoming college freshman. The following priorities were to be rated in the survey:

- Training for a career
- Relaxing and enjoying life before joining the real world
- Figuring out what I want to do in life
- Exploring and developing intellectually
- Developing important skills

Those five priorities were to be asked in a random order with a random pairing in order to eliminate any possible ordering bias. The pairings would allow the student to use an on-screen slider mechanism to rate one priority compared to another on a symmetric scale from 1 to 9. 1 would indicate that the priorities had equal importance and 9 would indicate that one priority is 9 times as important as the other. For example, one survey question may ask the student, “What is more/most important for you to get from college?”, where their priority options were “Training for a Career” and “Figuring out What I Want to do in Life.” A visual of this example survey question is below:

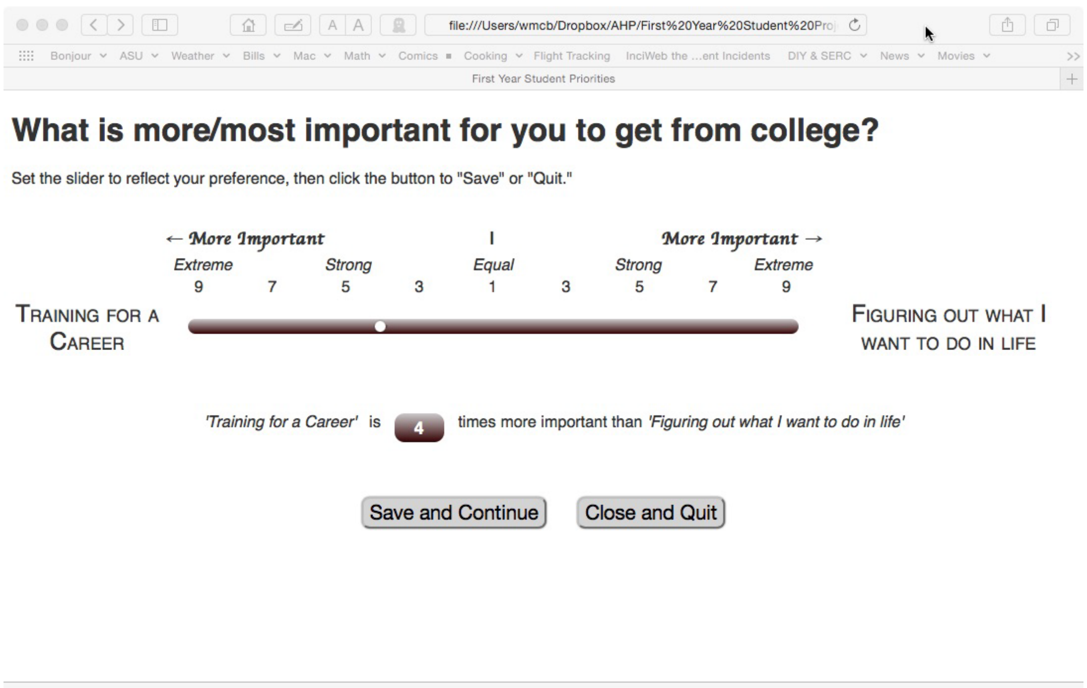

Figure 1: Sample Survey Question

Once enough data had been collected from the survey on Mechanical Turk, we would be able to to properly place the data into a pairwise-comparison matrix for each individual student. Since our goal was to study the freshman student population as a whole though, we could find the geometric mean of each row-column placement of each student’s pairwise-comparison matrix to use in one representative pairwise-comparison matrix. The mathematics behind this is presented below:

Let the superscript k represent each specific student. Then their pairwise-comparison matrix would appear as follows [3]:

$$A^k = \begin{pmatrix} a^k_{1,1} & a^k_{1,2} & a^k_{1,3} & a^k_{1,4} & a^k_{1,5} \\ a^k_{2,1} & a^k_{2,2} & a^k_{2,3} & a^k_{2,4} & a^k_{2,5} \\ a^k_{3,1} & a^k_{3,2} & a^k_{3,3} & a^k_{3,4} & a^k_{3,5} \\ a^k_{4,1} & a^k_{4,2} & a^k_{4,3} & a^k_{4,4} & a^k_{4,5} \\ a^k_{5,1} & a^k_{5,2} & a^k_{5,3} & a^k_{5,4} & a^k_{5,5} \end{pmatrix}$$

Note that the geometric mean is defined as $\left(\Pi^k_{n=1} x_n\right)^{\frac{1}{k}}$. So the representative pairwise-comparison matrix of the entire sample of incoming college freshman would be defined as:

$$A_{GM} = \begin{pmatrix} \sqrt[k]{a^1_{1,1} \cdot a^2_{1,1} \cdots a^k_{1,1}} & \cdots & \sqrt[k]{a^1_{1,5} \cdot a^2_{1,5} \cdots a^k_{1,5}} \\ \vdots & \ddots & \vdots \\ \sqrt[k]{a^1_{5,1} \cdot a^2_{5,1} \cdots a^k_{5,1}} & \cdots & \sqrt[k]{a^1_{5,5} \cdot a^2_{5,5} \cdots a^k_{5,5}} \end{pmatrix}$$

Unfortunately, we did not have enough time to complete the full institutional review board process and so we needed to revise our plan.

3.2 Revised Project

Since we still wanted to study how the Analytic Hierarchy Process can be implemented, the new goal of the project became to show that the Analytic Hierarchy Process is a feasible method in determining the implicit priorities of incoming college freshman. Therefore, a simulation was appropriate.

Since we wanted the simulation to demonstrate similar features to those of the actual survey, we randomized priority pairs by using a binomial distribution. The binomial distribution allows the priority pairs to be selected on a success/fail basis where the selections are independent and with replacement. This prevents any bias that may originate from the order that the priority options are given in. We then used a uniform distribution to choose beta paramters that would indicate which priority had more importance. By choosing beta parameters, the simulated student is likely to have a preference for one priority versus another. This again, makes the simulation much more realistic.

Once the initial setup of the simulation was complete, we simulated the responses of 1,000 students and placed them into a pairwise-comparison matrix. Just as before, we would be able to compile all 1,000 pairwise-comparison matrices into one representative matrix by taking the geometric mean of each row-column value. We could then continue the Analytic Hierarchy Process to find the representative positive eigenvector that would then give the overall ranking of priorities.

3.3 Analyzing My Own Priorities

To further investigate the Analytical Hierarchy Process with specific examples, I decided to perform the Analytic Hierarchy Process to see what my rankings were as an incoming freshman compared to my rankings now. I used a similar process of comparing priorities in pairs, placing my ratings in a pairwise-comparison matrix, and finding the eigenvector that implied my overall rankings. The only significant difference here is that the representative matrix found by using the geometric mean is not necessary since I only had one pairwise-comparison matrix for analyzing my priorities as a freshman and one pairwise-comparison matrix for analyzing my current priorities.

4 Results

4.1 Simulation Results

Recall that the simulation involved simulating the responses of 1,000 students. The following is an example of one student's responses placed into a pairwise-comparison matrix, where the first row and column represents training for a career, the second row and column represents relaxing and enjoying life, the third row and column represent figuring out what to do in life, the fourth row and column represent exploring intellectually, and the fifth row and column represent developing important skills.

$$\text{Student } k\text{'s response} = \begin{pmatrix} 1 & 1 & 1 & 1/4 & 5 \\ 1 & 1 & 1/3 & 3 & 7 \\ 1 & 3 & 1 & 1 & 7 \\ 4 & 1/3 & 1 & 1 & 4 \\ 1/5 & 1/7 & 1/7 & 1/4 & 1 \end{pmatrix}$$

Just as we had planned to do in the initial project, we compiled all of the pairwise-comparison matrices into one representative pairwise-comparison matrix by taking the geometric mean of each row-column value. The following is the representative matrix from the simulation:

$$\text{GM of Response} = \begin{pmatrix} 1 & 0.967 & 0.970 & 1.055 & 1.077 \\ 1.035 & 1 & 0.961 & 0.984 & 0.922 \\ 1.030 & 1.041 & 1 & 0.945 & 0.945 \\ 0.948 & 1.016 & 1.058 & 1 & 1.021 \\ 0.929 & 1.085 & 1.059 & 0.980 & 1 \end{pmatrix}$$

From the representative pairwise-comparison matrix, we were able to find the corresponding eigenvector:

$$\text{Eigenvector} = \begin{bmatrix} 0.453 \\ 0.438 \\ 0.443 \\ 0.450 \\ 0.451 \end{bmatrix}$$

From the eigenvector above, we can see that the overall ranking of the priorities are:

1. Training for a career
2. Developing important skills
3. exploring and developing intellectually
4. Figuring out what to do in life
5. Relaxing and enjoying life before joining the real world

4.2 Results from Analyzing My Own Priorities

Before actually conducting the Analytic Hierarchy Process on my own priorities as an incoming college freshman, I wanted to make a guess of what my rankings would be so that I could compare them with the results. My guessed rankings were:

1. Training for a career
2. Figuring out what to do in life

3. Developing important skills
4. Exploring and developing intellectually
5. Relaxing and enjoying life before joining the real world

When I actually compared each pair of priorities, I found that training for a career is 7 times as important as relaxing and enjoying life, twice as important as figuring out what I want to do in life, 4 times as important as exploring intellectually, and 4/3 as important as developing important skills. I also found the relaxing and enjoying life is 1/4 as important as figuring out what I want to do in life, 1/2 as important as exploring intellectually, and 1/7 as important as developing skills. Figuring out what I wanted to do in life was twice as important as exploring intellectually, and 3 times as important as developing important skills. Finally, exploring intellectually was 1/4 as important as developing skills. Therefore, my pairwise comparison matrix was the following:

$$\text{My Freshman Priorities} = \begin{pmatrix} 1 & 7 & 2 & 4 & 4/3 \\ 1/7 & 1 & 1/4 & 1/2 & 3 \\ 1/2 & 4 & 1 & 2 & 3 \\ 1/4 & 2 & 1/2 & 1 & 1/4 \\ 3/4 & 7 & 1/3 & 4 & 1 \end{pmatrix}$$

From the above matrix, I obtained the following eigenvector:

$$\text{Eigenvector} = \begin{bmatrix} 0.6807 \\ 0.0874 \\ 0.5449 \\ 0.1672 \\ 0.4517 \end{bmatrix}$$

Thus, the ranking of my priorities as an incoming college freshman, derived from the Analytic Hierarchy Process are:

1. Training for a career
2. Figuring out what I want to do in life
3. Developing important skills
4. Exploring and developing intellectually
5. Relaxing and enjoying life before entering the real world

Notice that the initial guessed rankings match the rankings that the Analytic Hierarchy Process gave.

Before evaluating my current priorities with the Analytic Hierarchy Process, I again wanted to predict my actual rankings. My guessed ranking of current priorities as an incoming graduate student is:

1. Train for a career
2. Exploring and developing intellectually

3. Figuring out what I want to do in life

4. Developing important skills

5. Relaxing and enjoying life before entering the real world

Notice that my current priorities are now listed in a slightly different order than my priorities as an incoming college freshman. I now view training for a career as 10 times as important as relaxing and enjoying life, 7 times as important as figuring out what to do in life, 1/2 as important as exploring intellectually, and 4 times as important as developing skills. I also view relaxing and enjoying life as 1/8 as important as figuring out what to do in life, 1/6 as important as exploring intellectually, and 1/3 as important as developing skills. Figuring out what I want to do in life is 1/2 as important as exploring intellectually and twice as important as developing skill. Finally, exploring intellectually is 4 times as important as developing skills. Therefore, the pairwise-comparison matrix for my current priorities is:

$$\text{My Current Priorities} = \begin{pmatrix} 1 & 10 & 7 & 1/2 & 4 \\ 1/10 & 1 & 1/8 & 1/6 & 1/3 \\ 1/7 & 8 & 1 & 1/2 & 2 \\ 2 & 6 & 2 & 1 & 4 \\ 1/4 & 3 & 1/2 & 1/4 & 1 \end{pmatrix}$$

From the above matrix, I obtained the following eigenvector:

$$\text{Eigenvector} = \begin{bmatrix} 0.7166 \\ 0.0567 \\ 0.2555 \\ 0.6310 \\ 0.1407 \end{bmatrix}$$

Therefore, according to the eigenvector above, the Analytic Hierarchy Process gave me the following ranking of current priorities:

1. Training for a career

2. Exploring and developing intellectually

3. Figuring out what I want to do in life

4. Developing important skills

5. Relaxing and enjoying life before joining the real world

Again, notice that the initial guessed ranking of current priorities matched the ranking derived from the Analytic Hierarchy Process.

5 Conclusion

5.1 Simulation Conclusions

Recall that the overall representative pairwise-comparison matrix derived from the geometric mean appears as follows:

$$\text{GM of Responses} = \begin{pmatrix} 1 & 0.967 & 0.970 & 1.055 & 1.077 \\ 1.035 & 1 & 0.961 & 0.984 & 0.922 \\ 1.030 & 1.041 & 1 & 0.945 & 0.945 \\ 0.948 & 1.016 & 1.058 & 1 & 1.021 \\ 0.929 & 1.085 & 1.059 & 0.980 & 1 \end{pmatrix}$$

Notice that every value in the matrix is close to 1. This makes sense due to the fact that it came from a simulation of random students. If these values were not close to 1, there might be some bias in our 'random' simulation. Also recall that the eigenvector found from that matrix is the following:

$$\text{Eigenvector} = \begin{bmatrix} 0.453 \\ 0.438 \\ 0.443 \\ 0.450 \\ 0.451 \end{bmatrix}$$

Notice that all values in the eigenvector are very close to one another. Again, this makes sense due to our simulation of random students with random responses.

The simulation demonstrates exactly how the Analytic Hierarchy Process can be used to analyze real data on the implicit priorities of incoming college freshman. By successfully completing the simulation, we know how to use the Analytic Hierarchy Process in this situaion and that the Analytic Hierarchy Process is an effective tool in analytically ranking priorities.

For the future, Danielle Kane will conduct the original survey at DePauw University once there is approval from the Institutional Review Board. Once enough data is collected, we may repeat the Analytic Hierarchy Process on the real data and continue to study any possible patterns or groupings of students and priority rankings.

5.2 Conclusions from my own Priorities

Recall that both times I performed the Analytic Hierarchy Process on my priorities as an incoming college freshman and on my current priorities as an incoming graduate student I found that the resulting ranking of priorities matched the guessed ranking of priorities. Even though my ratings of priorities had changed with respect to one another, the Analytic Hierarchy Process still produced a result that made sense. This heightened my confidence in the decision making tool and convinced me even more so that the process is feasible for the project of researching the overall ranking of priorities for incoming college freshman.

6 Appendix

The following will appear on the initial web page of the survey.

Participant Information and Consent Form

Purpose

The purpose of this study is to examine the implied priorities of first-year college students.

Procedure

If you agree to be in this study, you will be asked to do the following:

- Set relative important scores using web-based forms for several comparative statements about the purpose of attending college.

The total time required to complete the study should be no more than 15 minutes. Participants will earn $0.xx through Mechanical Turk for completing the survey.

Benefits/Risks to Participant

Participants will help contribute to the body of knowledge in sociology and operations research. There are no foreseeable risks in participating.

Voluntary Nature of the Study/Confidentiality

Your participation in this study is entirely voluntary and you may refuse to complete the study at any point during the experiment, or refuse to answer any questions with which you are uncomfortable. You may also stop at any time. Your name or any identifying information will never be connected to any results or to your responses on the survey; only aggregate data will be collected. Information that would make it possible to identify any participant will never be included in any sort of report. The data will be accessible only to those working on the project and will be erased at the completion of the analysis.

Your consent will be given by clicking the *Submit* button on the final web page of the survey. You may discontinue participation at any time by closing the web page; in that case, no data will be collected from any of your responses.

Contacts and Questions

If you have any questions regarding this study, contact Prof Wm C Bauldry at BauldryWC@appstate.edu or Prof Danielle Kane at daniellekane@depauw.edu.

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Survey Question List

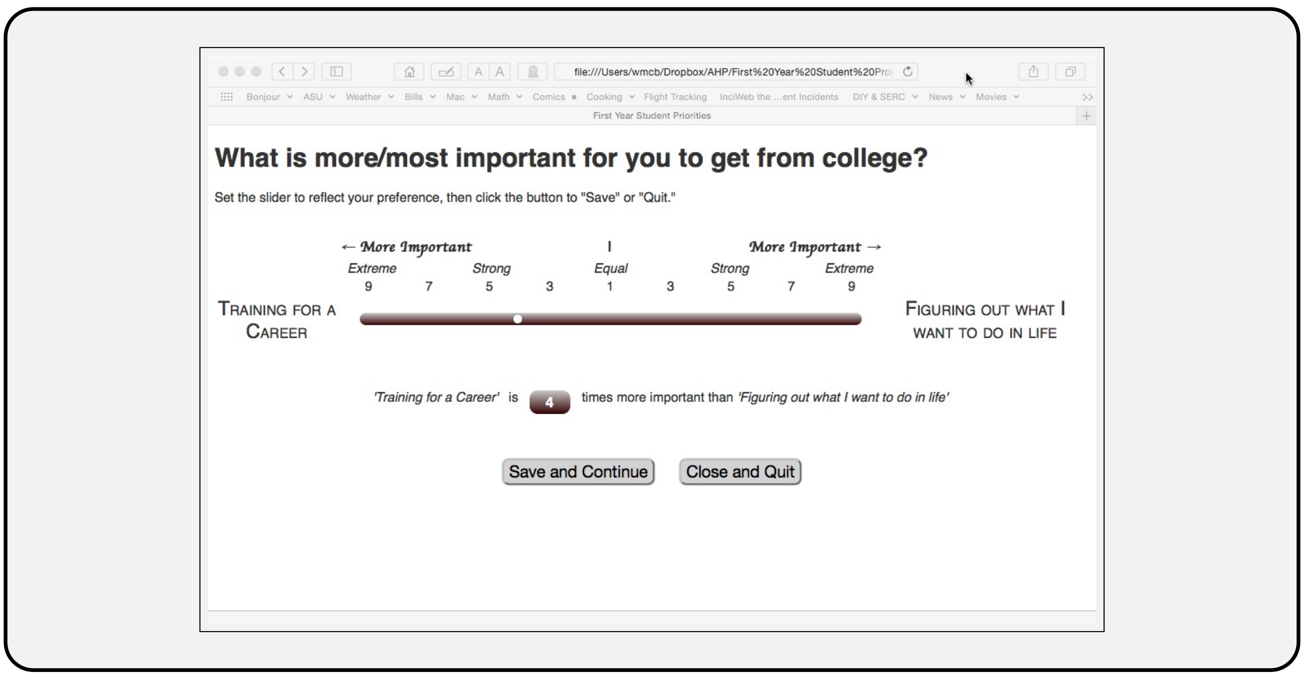
The question list we will use for participant comparisons is:

Students: What is more/most important for you to get from college?

1. Training for a career
2. Relaxing and enjoying life before joining the “real world”
3. Figuring out what I want to do in life
4. Exploring and developing intellectually
5. Developing important skills (eg. writing, manipulating data, public speaking)

Survey Page Layout

A typical question page in the web-based survey will be structured as in the following.



```
# Load Packages
interface(rtablesize = 20):
with(RandomTools):
with(Statistics):

# Set up randomized survey answers
RV := v -> RandomVariable(('Beta')(v[1],v[2])):
Scale := v -> map(x -> floor(1+8*x), v):
AddFlavor(Choice=rand(1..2)):
AddFlavor(B_params=rand(1..3)):
Questions := ["Training for a career",
    "Relaxing and enjoying life before joining the 'real world'",
    "Figuring out what I want to do in life",
    "Exploring and developing intellectually",
    "Developing important skills"]:
N := nops(Questions):
M := binomial(N, 2):

# Simulate a student response
SimResponse := proc(M)
    local SimStudent, Response, i;
    SimStudent :=
        Vector(M, Generate([3*Choice, [B_params,B_params]], makeproc=true))/3;
    Response := Vector(M);
    for i to M do
        Response[i] := [SimStudent[i], Scale(Sample(RV(SimStudent[i][2]),1))]
        end do;
    return(Response);
    end proc:

# Question pairs and indexing function
QPairs := [seq(seq([Questions[i], Questions[j]],j=i+1..N), i=1..N-1)]:
ndx := j -> 1+floor(j/5)+floor(j/8), j+1-3*floor(j/5)-2*floor(j/8)+2*floor(j/10):

# Simulate the Survey
NumberStudents := 1000:
for j to NumberStudents do
    rs := SimResponse(10);
    H := Matrix(N,N) + LinearAlgebra[IdentityMatrix](N);
    for i to 10 do
        H[ndx(i)] := rs[i][2][1]^((-1)^(3-rs[i][1][1]));
        H[ndx(i)[2],ndx(i)[1]] := 1/H[ndx(i)]
    end do:
    Results[j] := [rs,H]
end do:
```

Listing 1: Maple Simulation Code

7 Acknowledgements

I would like to formally thank my advisor, Dr. Bauldry, and my partners, Holly Creed and Danielle Kane, for their hard work and many contributions on this project. I would also like to recognize the Department of Mathematical Sciences for making this possible and giving me the opportunity to study and grow at Appalachian State University.

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