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Data Envelopment Analysis of First-Year Courses at Appalachian State University

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Overview

The MathSci O.R. Team

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Abstract

Data Envelopment Analysis (DEA) is a powerful technique that provides relative comparisons of very different types of depts in an organization. We adapt DEA to analyze distinct activities, rather than departments. We apply DEA to assess relative efficiency of large-enrollment, first-year courses, identifying resource inputs and activity outputs to investigate. Our methodology can be easily adapted to assess relative performance and/or efficiencies of resource utilization for activities across projects in industry and government.

For More Information

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The Project

Analyze required, first-year, general education classes at Appalachian. We assess four large-enrollment course blocks:

I. UCO 1200 First Year Seminar
II. BIO 1101 Bio in Society I, BIO 1102 Bio in Society II
III. ENG 1000 Expos Writing
IV. MAT 1010 Intro to Math, MAT 1020 College Algebra, MAT 1025 Precalculus

comparing resource utilization to multiple output production measures. DEA allows us to produce a ranking index based on several diverse inputs and outputs.

The Software

We use the 'Optimization' and 'simplex' packages of Maple™ 16, a computer algebra system, to perform the many simplex algorithm runs required by DEA.

(*Visit* www.maplesoft.com *for information about Maple.*)

Background: Data Envelopment Analysis

Charnes, Cooper, and Rhodes invented DEA in 1978, basing the technique on "combustion engineering's 'efficiency is the ratio of the actual amount of heat liberated . . . to the maximum amount which could be liberated'."

DEA Strengths

- Multiple input and multiple output models
- Comparison against all combinations of peers
- Inputs and outputs can have very different units

DEA Weaknesses

- Extreme point technique—noisy data causes significant error
- Estimates relative, not absolute efficiency
- Computationally intensive

DEA Linear Program Formulations

For each *Decision Making Unit* DMU_o ($o = 1..n$), with m inputs $[x_{io}]$ and s outputs $[y_{ro}]$, define the *relative efficiency* θ_o^* by

Input-oriented LP

$$\theta_o^* = \min_\lambda \theta$$

subject to

$$\sum_{j=1}^{n} x_{ij}\lambda_j \leqslant \theta\, x_{io},\ i = 1..m$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j \geqslant y_{ro},\ r = 1..s$$

Output-oriented LP

$$\phi_o^* = \max_\lambda \phi$$

subject to

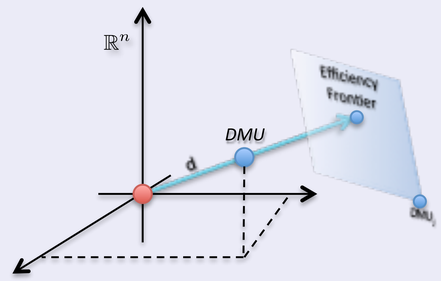
$$\sum_{j=1}^{n} x_{ij}\lambda_j \leqslant x_{io},\ i = 1..m$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j \geqslant \phi\, y_{ro},\ r = 1..s$$

with all $\lambda_j \geqslant 0$.

Data Envelopment Analysis Diagrammed

The *efficiency index* of a *decision making unit* is the relative distance



from the DMU to the *efficiency frontier*. The efficiency frontier is defined to be the convex hull of the DMU data points.

Project Input and Output Factors

Input Factors

- Number of students enrolled
- Number of sections/classes scheduled
- Number of seats offered
- Beginning student/teacher ratio
- Student credit hours generated
- Cost (estimate)

Output Factors

- Percent students successfully completing
- Number of students dropping/withdrawing
- Mean course GPA factor
- Ending student/teacher ratio
- Average cost per successful student (estimate)

Results and Conclusions

Complete Factor Set

- Leads to a convex hull with all data points near the frontier — no discrimination among DMUs

Subsets of Factors

- UCO at 66.9% efficiency index in relation to size inputs versus success & cost outputs; other DMUs at 100%
- BIO at 72.9% efficiency index in relation to credit hour inputs versus grade & success outputs; other DMUs at 100%

Further Directions

Incorporate more granular assessments by:

- Extending current blocks; e.g. extending first-year mathematics to all 'Quantitative Literacy' courses
- Adding each of the four required *Perspectives:* 'Aesthetic,' 'Historical & Social,' 'Local to Global,' and 'Science Inquiry'
- Incorporating further, more detailed, cost factors

References

1. Cooper, Seiford, and Zhu, eds, *Handbook on Data Envelopment Analysis*, Kluwer Acad Pub, Boston, 2004.
2. Ramanathan, *An Introduction to Data Envelopment Analysis: A Tool for Performance Measurement*, Sage Pub, 2003.