Algebra I Contest
Spring 2009

Directions:
Please read carefully. Do not begin the test until you are told to do so.

This test has two parts:
- **Part I** consists of 30 multiple-choice items.
  - You will have **1 hour** to work on Part I.
  - Only one answer is correct for each problem.
  - Answers must be placed on the answer sheet to receive credit.
  - There is no penalty for guessing.
- **Part II** consists of three tie-breaking essay questions.
  - You will have **15 minutes** to work on Part II.
  - When asked to do so, you will write your name on the paper and begin work.
  - The *work* and *support* for your solutions, not just your answers, will be graded; so show all of your work in a neat and organized manner.

Calculators (up to and including a TI-84) are allowed on both parts of the test.

Do not turn this page until you are asked to do so.
1. Evaluate the following expression: 
\[
4\left[(17 - 10)^3 + \left(\frac{17 + 200}{31}\right)\sqrt{36}\right] - 3^2(7 + 4) + \frac{22(9)^2}{18}
\]

a. 14  
b. \(\frac{35}{3}\)  
c. 11.7  
d. \(\frac{92}{3}\)  
e. 46.7

2. Which segment below gives a visual representation of how big \(\sqrt{10}\) is? (Assume that the distance between horizontally adjacent dots and vertically adjacent dots is 1 unit.)

- A. Segment AB
- B. Segment CD
- C. Segment EF
- D. Segment GH
- E. None of the above

3. Amanda received $500 from her grandparents on her birthday this year. They told her she had to put it into a savings account and could not touch it until she was 22 years old. Amanda turned 12 years old on her birthday. She went immediately and put her money in a savings account that gains interest at a rate of 6.5% compounded annually. How much money will she have accrued in interest from her savings account if she adds no other money to the account and she withdraws her money on her 22nd birthday?

- A. $438.57
- B. $325.00
- C. $1498.30
- D. $825.00
- E. $938.57

4. Find the equation of the vertical line that intersects the two functions \(-x + 2y = -13\) and \(10x - 3y = 62\) at their point of intersection.

- A. \(x = 5\)
- B. \(x = -4\)
- C. \(y = 5\)
- D. \(y = -4\)
- E. none of the above
5. Which solution set best describes the graph below:

-3 -2 -1 0 1 2 3

a. \( x \leq -\frac{2}{10} \) and \( x > \frac{13}{10} \)

b. \( x \leq \frac{2}{10} \) and \( x \geq \frac{13}{10} \)

c. \( x \leq \frac{2}{10} \) or \( x > \frac{13}{10} \)

d. \( x \leq -\frac{2}{10} \) or \( x \geq \frac{13}{10} \)

e. \( x \leq -\frac{2}{10} \) or \( x > \frac{13}{10} \)

6. If \( a_1 = 100000 \) is the first term of this sequence, which number below best describes the value you would need to divide 6250 by in order to get the 13th term in the sequence?

100000, 50000, 25000, 12500, 6250, ...

a. 2
b. \( 2^8 \)
c. 8
d. \( 2^{13} \)
e. 256.14

7. If \( y \) varies directly as \( x \) and \( y = 46 \) when \( x = 2 \frac{3}{10} \), find what \( x \) must be when \( y = 42 \).

a. \( x = -2 \frac{3}{10} \)

b. \( x = 21 \)
c. \( x = 20 \)
d. \( x = 2 \frac{1}{10} \)
e. \( x = -23 \)
8. The equation \( \frac{1}{f} = \frac{1}{a} + \frac{1}{b} \) is used in designing cameras and optical equipment. Kimo was working in the lab on a lens and needed to solve the equation for \( f \). What is the correct equation written in terms of \( f \)?

   a. \( f = a + b \)
   b. \( f = \frac{a + b}{ab} \)
   c. \( ab = fa + fb \)
   d. \( f = \frac{ab}{a + b} \)
   e. \( f = ab \)

9. On Mother's Day, a local flower shop owner sold roses for $45/dozen. The day after the holiday, he realized that he had not sold as many roses as he had planned, so he decided to mark the price down by 15% each day. When is the first day that the price per dozen roses is $12 or less?

   a. 10 days
   b. 9 days
   c. 8 days
   d. 7 days
   e. none of the above

10. Completely factor the following polynomial expression:

\[
6z^2(1-z^2) - 5z(1-z^2) + z^2 - 1
\]

   a. \( z(1-z^2)(6z-5) + z - 1 \)
   b. \( (1-z^2)(6z+1)(z-1) \)
   c. \( (6z-1)(z+1)^2(z-1) \)
   d. \( -(6z+1)(1+z)(1-z)^2 \)
   e. \( (6z^2 - 5z - 1)(1 - z^2) \)

11. Given the following right triangle, find the value of \( x \) if the expressions given in the diagram represent the side lengths of the triangle.

   a. 0  b. 4  c. -30  d. 30  e. none of the above
12. Many factors can influence how high a BMX dirt-bike racer can jump in a race. One of those is the weight of the bike. The table below gives the maximum jump height for various bike weights.

<table>
<thead>
<tr>
<th>WEIGHT (in pounds)</th>
<th>HEIGHT (in inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>10.35</td>
</tr>
<tr>
<td>19.5</td>
<td>10.3</td>
</tr>
<tr>
<td>20</td>
<td>10.25</td>
</tr>
<tr>
<td>20.5</td>
<td>10.2</td>
</tr>
<tr>
<td>21</td>
<td>10.1</td>
</tr>
<tr>
<td>22</td>
<td>9.85</td>
</tr>
<tr>
<td>22.5</td>
<td>9.8</td>
</tr>
<tr>
<td>23</td>
<td>9.79</td>
</tr>
<tr>
<td>23.5</td>
<td>9.7</td>
</tr>
<tr>
<td>24</td>
<td>9.6</td>
</tr>
</tbody>
</table>

If you use a best-fit line to model this data, which statement below best describes the relationship between bike weight and maximum jump height?

a. For every 1-pound increase in weight, the height decreases \( \frac{15}{100} \) of an inch.

b. For every 1-inch increase in height, the weight decreases \( \frac{15}{100} \) of a pound.

c. For every 1-inch increase in height, the weight increases \( \frac{15}{100} \) of a pound.

d. For every \( \frac{1}{2} \)-pound increase in weight, the height decreases 0.05 inches.

e. none of the above

13. Find an equation of a line that goes through the point \((8, 0)\) and does not intersect the line in the graph below.

a. \(-5x + y - 12 = 0\)

b. \(-5x + y - 40 = 0\)

c. \(-4x + y + 32 = 0\)

d. \(x - \frac{1}{5}y + 3 = 0\)

e. \(-5x + y + 40 = 0\)
14. Which system of equations below has (4,3) as a solution?
   a. $4x + 3y = 25$
      $4x + 3y = 12$
   b. $5x + y = 23$
      $x + 5y = 19$
   c. $x^2 - 2x - y = 5$
      $x - 2y = -2$
   d. Both A and B
   e. Both B and C

15. Robbie has $20 in a checking account that accrues no interest. He just got a job that pays $800 a month. If his monthly expenses are $550 and he has no other source of income, what is the maximum amount of money that Robbie could save in his account at the end of 2 years?
   a. $6020.00
   b. $6000.00
   c. $1620.00
   d. $520.00
   e. $500.00

16. The golden ratio can be illustrated by a segment whose length is $x + y$ where $x$ and $y$ satisfy the proportion:

$$\frac{x}{y} = \frac{x + y}{x}$$

Find what $y$ would be in inches for a segment that satisfies this proportion and has $x = 1$ inch.
   a. $y = 1 + \frac{1}{y}$
   b. $y = \frac{1 - \sqrt{5}}{2}$
   c. $y^2 + y = 1$
   d. $y = \frac{1 + \sqrt{5}}{2}$
   e. Both b and d
17. Wanda is having a party at a local restaurant and she is paying for all of her guests to eat. The restaurant has a deal where all of the guests can eat for much cheaper if there is one set menu. The menu price is $11.75 per guest (gratuity not included). If Wanda has $900 to spend and she is leaving 18% gratuity, how much money will she have left over if she invites the maximum number of guests for her $900?

a. $0.00  
   b. $11.00  
   c. $12.32  
   d. $12.64  
   e. none of the above

18. You are given a square that is x feet by x feet. You make a new figure by increasing the length of top and bottom sides of your square by 1 foot and decreasing the length of right and left sides of your square by 1 foot. What will be the relationship between the area of your original square and the area of the new figure you just made?

a. The areas will be the same.

b. The area of the square will be 1 square foot bigger than the area of the new figure.

c. The area of the new figure will be 1 square foot bigger than the area of the square.

d. The area of the new figure will be \((x + 1)^2\) square feet bigger than the area of the square.

e. The area of the new figure will be \((x - 1)^2\) square feet bigger than the area of the square.

19. If a and b are constants, then the graph of which recursively defined sequence below would best be modeled by a linear function?

a. 
   \[
   \begin{align*}
   u(0) &= a \\
   u(n) &= u(n-1) \times b + 1
   \end{align*}
   \]

b. 
   \[
   \begin{align*}
   u(0) &= a \\
   u(n) &= u(n-1) + b
   \end{align*}
   \]

c. 
   \[
   \begin{align*}
   u(0) &= a \\
   u(n) &= u(n-1) + bn
   \end{align*}
   \]

d. 
   \[
   \begin{align*}
   u(0) &= a \\
   u(n) &= u(n-1) \times b
   \end{align*}
   \]

e. 
   \[
   \begin{align*}
   u(0) &= a \\
   u(n) &= \frac{u(n-1)}{b}
   \end{align*}
   \]
20. Each function below represents the way a certain quantity \( Q \) changes over time \( t \). If \( a \) and \( b \) are constants, which function indicates that the quantity \( Q \) is not decreasing?

a. \( Q(t) = ab^t; a > 0, 0 < b < 1 \)
b. \( Q(t) = a + bt; a > 0, b < 0 \)
c. \( Q(t) = ab^{-t}; a > 0, b > 1 \)
d. \( Q(t) = ab^t; a < 0, b > 1 \)
e. \( Q(t) = a + bt; a > 0, b = 0 \)

21. Campus Express offers college students summer charter trips in Europe. The company flies three kinds of airplanes: the Airbus 100, the Airbus 200, and the Airbus 300. Each plane is outfitted with tourist, economy, and first-class seats. The number of each kind of seat in the three types of planes is shown in Matrix A below. Campus Express expanded its fleet to four of each kind of airplane. Matrix B lists the total number of passengers that flew (by plane type) on Campus Express on July 2\textsuperscript{nd} of last year. Which matrix below represents the number of seats Campus Express did not sell in its fleet of planes on that day?

\[
\text{Matrix A} = \begin{bmatrix}
50 & 75 & 40 \\
30 & 45 & 25 \\
32 & 50 & 30
\end{bmatrix}
\]

\[
\text{Total Number of Passengers (by plane and seat type) for July 2, 2008} = \begin{bmatrix}
197 & 261 & 160 \\
118 & 100 & 73 \\
64 & 111 & 54
\end{bmatrix}
\]

\[
\begin{bmatrix}
147 & 186 & 120 \\
88 & 55 & 48 \\
32 & 61 & 24
\end{bmatrix}
\]

a. \[
\begin{bmatrix}
147 & 186 & 120 \\
88 & 55 & 48 \\
32 & 61 & 24
\end{bmatrix}
\]

d. \[
\begin{bmatrix}
3 & 39 & 0 \\
2 & 80 & 27 \\
64 & 89 & 66
\end{bmatrix}
\]

\[
\begin{bmatrix}
397 & 561 & 320 \\
238 & 280 & 173 \\
192 & 311 & 174
\end{bmatrix}
\]

b. \[
\begin{bmatrix}
397 & 561 & 320 \\
238 & 280 & 173 \\
192 & 311 & 174
\end{bmatrix}
\]

e. \[
\begin{bmatrix}
200 & 300 & 160 \\
120 & 180 & 100 \\
128 & 200 & 120
\end{bmatrix}
\]

c. \[
\begin{bmatrix}
247 & 336 & 200 \\
148 & 145 & 98 \\
96 & 161 & 84
\end{bmatrix}
\]
22. In the diagram below, line AC intersects the circle, whose center is A, at two points, C and B. The coordinate of A is \((-\frac{19}{7}, \frac{10}{3})\) and the coordinate of C is \((\frac{30}{7}, \frac{-2}{3})\).

Find the coordinate of B.

a. \((-\frac{8}{7}, \frac{22}{3})\)

b. \((-\frac{8}{7}, \frac{18}{3})\)

c. \((-\frac{19}{14}, \frac{14}{6})\)

d. \((-\frac{68}{7}, \frac{18}{3})\)

e. \((-\frac{68}{7}, \frac{22}{3})\)

23. The parents of three brothers will not let their children go see a PG-13 movie if they are under the age of 13. The three brothers have ages that are consecutive even integers. The sum of their ages is 42. The oldest child goes to a movie every year on his birthday as part of his celebration. How many birthdays has the oldest child been allowed to see a PG-13 movie on his birthday?

a. 2  b. 3  c. 4  d. 12  e. 16

24. Which value(s) below best describes the solution set for the equation \(\frac{48x^2}{1 + 2x^{-1}} = 2\)

a. \(x = -4\)

b. \(x = \frac{1}{12}\)

c. \(x = 6, x = -4\)

d. \(x = -6, x = 4\)

e. \(x = -6\)
25. What is the largest coefficient when the following expression is simplified?

\[(x^2 + \frac{2x^2}{x^{19}})^5 - (3x^3)(4x^4) + (11x)^2\]

a. 10    b. 12    c. 32    d. 121    e. 243

26. Two cyclists are competing in a 12-mile race on a quarter-mile circular track. They begin at the same time. One biker’s average speed is 24 mph. The other’s average speed is 30 mph. How many laps will the slower cyclist have left to complete after the faster cyclist finishes?

a. 2.4    b. 3    c. 9    d. 9.6    e. 12

27. If \(Ax + By + C = 0\), what does the value of \(C\) tell you about the graph of this equation?

a. The graph crosses the vertical axis at \(x = -C\).

b. One of the points on the graph is \((0, -\frac{C}{B})\).

c. The graph increases at a rate of \(C\).

d. The graph decreases at a rate of \(\frac{C}{B}\).

e. The graph has a zero (or root) at \(x = C\).

28. Ohio has half the population of California, and twice the population of Indiana. If the total population of the three states is 38,500,000, find the population of Indiana.

a. 2,750,000

b. 5,500,000

c. 11,000,000

d. 22,000,000

e. Not enough information is given to find the solution.
29. An Algebra I student found her *best-fit* linear equation for $M$ the number of watercraft-related manatee deaths $t$ years since 1974. Her model’s prediction for 1974 equaled the actual number of watercraft-related manatee deaths, which was 3 deaths. The actual number of deaths in 1990 was 47 deaths. Her model underestimated that number by 2 deaths. Find the student’s best-fit linear equation $M(t)$.

a. $M(t) = 2.75t + 3$

b. $M(t) = 2.625t - 5178.75$

c. $M(t) = 2.625t + 3$

d. $M(t) = 2.75t - 5425.5$

e. $M(t) = 3t + 2.625$

30. The graph of a quadratic function crosses the horizontal axis at $x = -5$ and $x = 15$. At what $x$ value does this function reach its maximum output value?

a. At $x = 5$

b. At $x = 10$

c. At $x = 20$

d. It never reaches a maximum output value.

e. Not enough information is given to tell for sure.
Algebra I Contest
March 25, 2009
Tie Breaker Questions – 15 minutes time limit
(Be sure to show all work. You will be given points for your process as well as your answer.)

1. A three-digit number increases its value by 9 if we exchange the second and third digits and it increases its value by 90 if we exchange the first and second digits. By how much will the three-digit number’s value increase if we exchange the first and third digits?
2. If you raise 3 to the 330\textsuperscript{th} power, what will be the ones (or units) digit of the result?
3. A certain number leaves a remainder of 1 when it is divided by 2, 3, 4, 5, or 6, but leaves no remainder when it is divided by 7. What is the smallest number that fits this description?