Guide to Preparing for the Comprehensive Exam

The comprehensive examination serves two purposes: (1) To assess your understanding of basic concepts in four key topics: Analysis, Linear Algebra, Modeling, and Statistics.

The courses covering this material and the fundamental idea that may be included on the exam are:

MAT 5610: Analysis I

- Understanding of continuity and uniform continuity
- Mastery of limit definitions for differentiation and integration
- Understanding of convergence and divergence of sequences and series
- Understanding of point-wise and uniform convergence and applications to integration

MAT 5230: Linear Algebra

- Mastery of computational skills from undergraduate linear algebra
- Understanding of theoretical concepts related to real and complex vector spaces and linear transformations
- Understanding of spectral theory and diagonalization
- Understanding of inner product spaces and orthogonality

MAT 5330: Mathematical Modeling

- Knowledge of the cycle and process of modeling
- Ability to create a mathematical formulation of a problem
- Ability to produce, interpret, analyze, and critique a formulation or solution
- Understanding of the roles sensitivity and uncertainty play in the modeling process

STT 5811: Statistical Concepts and Applications I

- Appropriate use of technology
- Appropriate presentation of graphics and descriptive statistics
- Understanding of probability and sampling distributions
- Understanding of statistical inference, including using correct procedures, and drawing proper conclusions

Note that these courses will cover more than the fundamental knowledge needed for the comprehensive exam.

Below are additional details regarding preparation for each component.

ANALYSIS

The analysis portion of the exam is completed in the seated portion of the exam.

There will be three sections. For each section you will select one problem from a list of several. The questions are intended to ask about your understanding of basic concepts and definitions, and not to require perfect recall of obscure minor technical results. You do need to remember some things. It is good to know

- major definitions (e.g. continuity of a function at a point, the limit definition of the derivative, point-wise convergence of a sequence of functions, uniform convergence of a sequence of functions, and so on)
- important theorems, especially theorems with names (e.g. intermediate value theorem, mean value theorem, Heine-Borel theorem, Bolzano-Weierstrass theorem, etc.).

The problems on your midterm and final exams in MAT 5610 are good examples of the types of problems you might see. In fact, that is a good place to start reviewing, making sure that you know how to fix any errors that cost you any points on those exams.

In addition to reviewing the exams, reviewing the text and notes and writing a list of the important definitions and theorems would be very helpful.

LINEAR ALGEBRA

1. Mastery of computational skills from undergraduate linear algebra. We really want to know that you understand these ideas; you may be teaching them one day soon! These skills include things like

- solving systems via row-reduction and talking about the process;
- multiplying matrices and talking about the process;
- finding bases for vector subspaces, particularly column and null spaces, and talking about the process;
- finding coordinate vectors and change of basis matrices and talking about the process;
- finding inverse matrices and talking about the process, etc.
- discussing the many equivalent conditions for invertibility of a square matrix
- calculating the determinant of a square matrix and talking about the geometric/algebraic significance of this number

Focus on both how to do these things and why what you are doing works.

2. Understanding of theoretical concepts related to real and complex vector spaces and linear transformations. Two ideas that you started thinking about in undergraduate linear algebra but that we really expanded on in graduate linear algebra are vector spaces/subspaces and linear transformations. Reviewing key ideas (including writing proofs) in these areas would be helpful. Ideas include

- applying definition of both linear transformation and vector subspace;
- linear independence/spanning sets/basis,
- matrix of a linear transformation with respect to various bases;
- injective/surjective linear transformations and kernel/range as well as connections between these ideas;
- inverse transformations and connections to matrices, etc.

Here it would be good to make sure you understand these concepts in R^n/C^n, in vector spaces other than R^n/C^n (like polynomial or matrix spaces), and in a general more theoretical sense.

Understanding of spectral theory and diagonalization. Another focus for our course was spectral theory (eigenvalues/eigenvectors). Again review basic definitions and make sure you can apply these ideas to linear operators on R^n/C^n, on vector spaces other than R^n/C^n (like polynomial or matrix spaces), and in a general, more theoretical sense. Remember that we worked closely from the definition but then used our understanding of the determinant and equivalent statements for invertibility to derive the characteristic equation. We were then able to find eigenvalues from the characteristic equation and view eigenspaces as certain null spaces. Reviewing key ideas (including writing proofs) in these areas would be helpful. Ideas include

- definition of eigenvalue/eigenvector/eigenspace and connections to the characteristic equation

- theoretical properties of eigenvectors -- e.g. eigenvectors with different eigenvalues are linearly independent
- diagonalizable matrices -- definitions, significance, process to diagonalize when possible
- applications of diagonalization--e.g. discrete dynamical system or Markov processes

Understanding of inner product spaces and orthogonality. We introduced the idea of an inner product space and a normed vector space.

- apply the definitions of (and able to write proofs about) general inner products, norms, and orthogonal subspaces.
- apply and explain the gram-schmidt orthogonalization process, although we would provide the formulas for this process on the exam, so please don't feel like you need to memorize those.
- decompose a space in terms of a subspace and its orthogonal complement
- work with (and explain) applications of orthogonal subspaces (e.g. least squares minimization problems)

MODELING

There are two parts to the modeling comp:

- A brief in-class portion with general questions about the modeling process. To prepare you should review the steps typically included in a discussion of the process, including typical ways of approaching assumptions and mathematical formulation. Questions in the past have included discussions of discrete versus continuous, dynamic versus static, stochastic versus deterministic, Occam's razor, outlining assumptions, the process as a cycle, evaluation and reflection, and others.
- A take-home project for you to complete. Here we will review your work to judge your understanding of and/or ability to complete the following:
    - The process, including the iterative nature of modeling.
    - Creation and solution of mathematical model(s), including why you chose your model and solution approaches.
    - A discussion of the strengths and weakness of your model(s).

The project is left as an open-ended problem, but it is expected that you will investigate the problem creatively and in depth rather than looking for a minimal model. Without constraining you too closely, we want to see what you can do and to make sure you understand the fundamentals of mathematical modeling. You should be prepared to model using a variety of approaches with appropriate computational tools and present the solution in a typeset format suitable for the audience outlined in the project.

STATISTICS

Seated Portion:

This will be a series of multiple choice, generally conceptual questions. You are allowed to explain your choices for any of the questions as needed. You should not need a calculator or statistical software but you are welcome to use it if needed. Conceptual topics that you should focus on would be:

- Summarizing the distribution of a quantitative variable (shape, center, spread, modality, outliers)
- How the shape of a quantitative variable affects measures of center and spread
- Concepts in bivariate analyses of quantitative variables (correlation, regression)
- Sampling Distributions, especially the Central Limit Theorem
- Statistical Inference:
    - Interval Estimates, Margin of Error
    - Hypothesis Testing: Hypotheses, p-values, decisions, types of errors

Take Home Portion:

The take home portion of the exam is a data analysis exercise, for which you will use statistical software to compute answers to the questions, and you will write your exam as a report involving computational output (and code, if a programming language was used) and written answers to the questions.
This part of the exam is much more like a homework set from Stt 5811, but will cover topics from the whole course. The exam explores one data set for a series of exercises. Be prepared to provide a descriptive statistics summary of multiple variables, including calculating summary statistics, and providing appropriate graphical summaries of one or more variables. Also be prepared to conduct various inferential procedures (interval estimates and hypothesis tests) for the data, and explain the conclusions clearly.

For the take home exam you are allowed to consult any resources of your choosing, including your course textbook, class notes, and online sources. Be prepared to comment on your code when it is non-standard, and to write clear explanations of your conclusions for all problems.